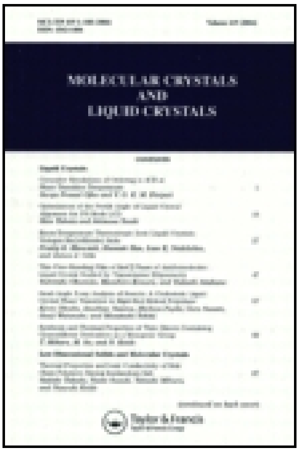

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

Computer Modelling of the Light Propagation through the Complex Anisotropy Systems - the way of a Determination of the Optical Parameters of the Liquid Crystal Displays

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Version of record first published: 18 Oct 2010

Mol. Cryst. Liq. Cryst., Vol. 375, pp. 441-454

1058-725X/02 $12.00 + .00

Computer Modelling of the Light Propagation through the Complex Anisotropy Systems – the way of a Determination of the Optical Parameters of the Liquid Crystal Displays

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A mathematical and numerical method for an analysis of a liquid crystal displays (LCD) has been presented. The assumptions of the calculations of a light propagation through the display are shown. The real conditions of a work of these displays have been taken into account. Basing of this mathematical model a computer program for a numerical modelling of the optical parameters of LC displays working in real conditions has been worked out. This program makes possible to obtain the most important parameters of a display such as: contrast ratio, luminance in on- and off-state, spectral characteristics of light and its colour coordinates in these both states. The results obtained from this program have been verified experimentally. Very high conformity of these results has been obtained.

Keywords numerical modelling, optical parameters, LCD

INTRODUCTION

The analysis of a light propagation through the LCD is very important for the determination of the optical parameters of the display for different applications. These parameters can be measured, but large quantity of the elements of the display (e.g. polarizing films, glass, conductive layer, liquid crystal layers), external parameters (e.g. illuminated light, sensitivity of human eye, possibility of two illuminated sources – external and internal) and the fact, that the display elements can have the different properties e.g. can be absorbed centre with isotropic (e.g. conductive layer) or linear absorption (e.g. polarizing films and LC layer, which are the dichroic centres) makes, that the determination of the optimal working conditions of the display is very difficult or even impossible by the measurements. In our opinion, only the computer program for numerical modelling of the display can make it possible. This program has to be worked out with the real conditions of the display operation assumption, yet. It means, that the corrected determination of the all display elements optical parameters must be done, the all optical phenomena must be taken into account and the optical parameters of the display must be correctly determined. Additionally, the measurement ways to determine these optical parameters of the display elements must be set unambiguously. Numerical modelling of LCDs has been used for more then 2 decades and the several commercial program are accessible [1,2]. Unfortunately, we never know all details of its structure, so the using of this programs for specific calculations can give sloppy results. Therefore, we have decided on working out us own software for us specific calculation (first of all for calculations of the static optical parameters for LCD with dichroic layers). In this work the measurement methods of the optical parameters of the display elements, the mathematical method to calculate of the light propagates through the real display and the results for different display operation conditions have been presented.

MODEL OF THE LCD AND THE MEASUREMENT METHODS

The following model of the LC display has been assumed:

- The display is constructed according to the Fig. 1. Only these elements have been taken into account in our calculations.
- The simplified Fabry-Perrot interference is taken into account (for the boundaries glass-conductive layer and conductive layer-liquid crystal) – according to Fig. 2. Additionally, the light phase changes in the case the reflections from boundaries of the absorption centres is taken into account.
- The polarising films are dirchroic centres. Their optical parameters are expressed by indices $T_p(II)$ and $T_p(+)$ which denote the linearly polarised light transmission (polarised according to the polarizer axis and perpendicularly to it, respectively) through single film. In

this same way, these coefficients (for analyser denoted as $T_a(II)$ and $T_a(+)$ are given for reflective film. In this case the measurements are made for reflected light (see Fig.3).

- The glass and the conductive layer have isotropic absorption.
- The liquid crystal layer is dichroic centre. The ellipsoid of the refractive and absorption indices is the same.
- The analyser has metallic mirror. The properties of this mirror are included in the analyser coefficients.
- The calculations are done according to modified GOA [3,4,5,6,7,8,9] (Geometry Optics Approximation) method. The co-ordinate systems assumed for the analysis are presented in Fig. 4.

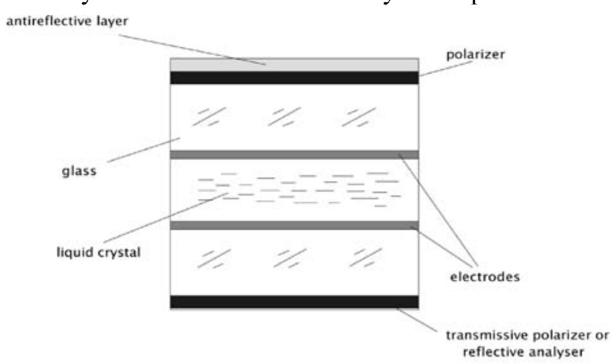

FIGURE 1 The scheme of the LCD assumed for the mathematical analysis.

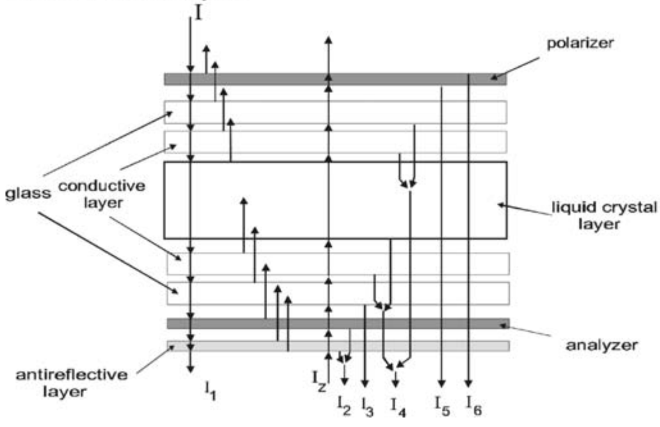

FIGURE 2 The scheme of the light propagation through the transmissive LCD. I denotes an intensity of the illuminating light

(from a source), I_Z - an intensity of the external light (from an environment), I_2, I_3, I_4, I_5 and I_6 – the intensities of the light reflected from the appropriate phase boundaries. For reflective display the light propagation have been analysed in the same way, but the transmissive polarizer has been replaced with reflective analyser. Additionally, in this case only the external source occurs.

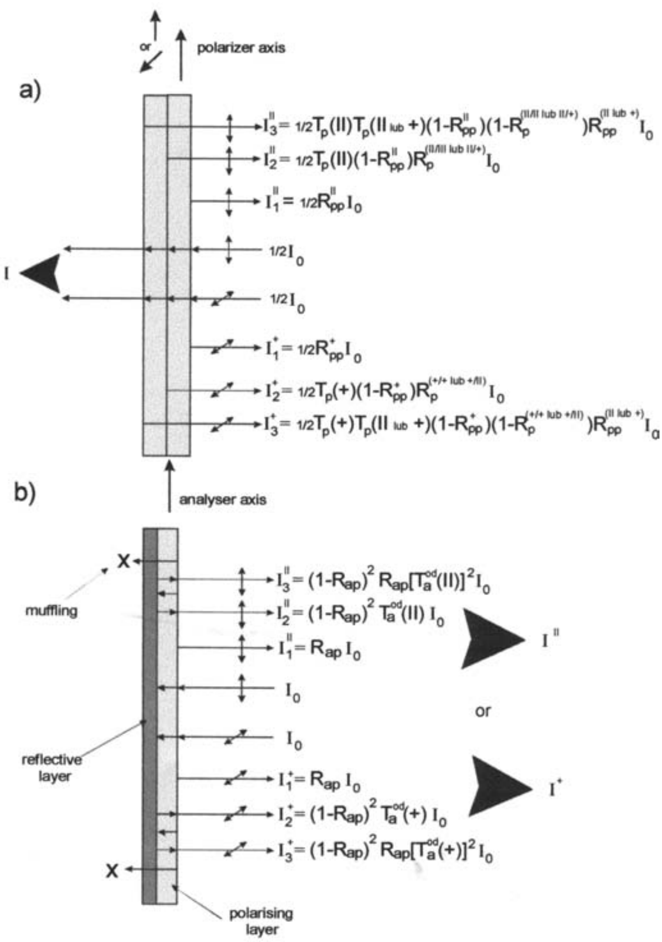

FIGURE 3 The measurement system of the light propagation through the polarising films:

a) for two parallel or perpendicular polarizers;
b) for the single reflective analyser.

The intensities I^{II} and I^{+} denote the reflected light intensity polarised according to the first polarizer axis and perpendicularly to it, respectively (Fig. 3a) and the light polarised according to the reflective analyser axis or perpendicularly to it (Fig. 3b). The light reflection coefficients R denote: $R^{II}{}_{pp}$ and $R^{+}{}_{pp}$ – the reflection coefficients for air-polarizer boundary for the light polarized according to the polarizer axis and perpendicularly to it, respectively, $R^{1/2}{}_{p}$ – the reflection coefficients for the light propagation through polarizer-polarizer boundary for the light linearly polarized, where the polarization direction (with reference to the first polarizer axis) is denoted by symbol *1* and second *2*, R_{ap} – the reflection coefficients from analyser-air boundary, $T_p(II)$, $T_p(+)$, $T_a(II)$ and $T_a(+)$ denote the real (without the reflection) transmission coefficients for the light polarised according or perpendicular to the polarizer or transmissive analyser axis, $T_a^{od}(II)$ and $T_a^{od}(+)$ denote the real "transmission" coefficients for the light polarised according or perpendicular to the reflective analyser axis.
The measurements done according Fig. 3 make possible to obtain the film polarising coefficient, precisely. In this case one can use the following formulas [10]:

$$\begin{aligned} WP &= \sqrt{\frac{T^{II}_{pm} - T^{+}_{pm}}{T^{II}_{pm} + T^{+}_{pm}}} && -\textit{for transmissive film} \\ WP^{od} &= \frac{T^{od\,II}_{am} - T^{od\,+}_{am}}{T^{od\,II}_{am} + T^{od\,+}_{am}} && -\textit{for reflective film} \end{aligned} \qquad (1)$$

where letter „*m*" denotes the value measured in measurement system according to the Fig. 3 and the symbols "*II*" and "*+*" denote two parallel and crossed polarizers (for transmissive film) and the light polarised according and perpendicularly to the analyser axis (for reflective film), respectively. Carrying out the measurements of the polarising film transmission the value of dichroic transmission coefficient (*T(II)* and *T(+)*) and the polarising coefficient WP are obtained. Additionally, the reflection coefficient value of the analysed film is obtained. For the reflective film the mirror properties are included in the transmission coefficient. It facilitates the calculation very much.
The separate problem is to determine the properties of the conductive layer. To make it easier the measurement of the non-polarised light transmission through the system analysed glass with conductive layer has been proposed. The equation taking into account reflections, Fabry-Perot interference and absorption can be described (2). From

this formula the conductive layer absorption coefficient can be obtained [11].

$$\frac{A\cdot\left(B+\alpha_{ITO}^{2}\right)\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}}{\left(C+\alpha_{ITO}^{2}\right)\cdot\left(D+\alpha_{ITO}^{2}\right)}\cdot\left\{\begin{array}{l}1+e^{-2\cdot\alpha_{ITO}\cdot d_{ITO}}\cdot\left[\dfrac{\left(E+\alpha_{ITO}^{2}\right)\cdot\left(F+\alpha_{ITO}^{2}\right)}{\left(C+\alpha_{ITO}^{2}\right)\cdot\left(D+\alpha_{ITO}^{2}\right)}\right]^{2}+\\+2\cdot e^{-\alpha_{ITO}\cdot d_{ITO}}\cdot\dfrac{\left(E+\alpha_{ITO}^{2}\right)\cdot\left(F+\alpha_{ITO}^{2}\right)}{\left(C+\alpha_{ITO}^{2}\right)\cdot\left(D+\alpha_{ITO}^{2}\right)}\cdot\cos\gamma\end{array}\right\}-T_{ITOm}=0$$

where:

$$A=\left[\frac{32\cdot\pi\cdot n_{g}}{\lambda\cdot\left(n_{g}+1\right)}\right]^{2};\quad B=\left[\frac{4\cdot\pi\cdot n_{ITO}}{\lambda}\right]^{2};\quad C=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}+n_{g}\right)}{\lambda}\right]^{2};$$

$$D=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}+1\right)}{\lambda}\right]^{2};\quad E=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}-n_{g}\right)}{\lambda}\right]^{2};\quad F=\left[\frac{4\cdot\pi\cdot\left(n_{ITO}-1\right)}{\lambda}\right]^{2};\qquad(2)$$

and:

$$\gamma=arc\,\mathrm{tg}\left(\frac{-8\cdot\pi\cdot\alpha_{ITO}}{\left(\lambda\cdot n_{ITO}\right)^{2}+4\cdot\pi\cdot\alpha_{ITO}^{2}-\lambda^{2}}\right)+$$
$$+arc\,\mathrm{tg}\left(\frac{-8\cdot\pi\cdot\alpha_{ITO}\cdot n_{g}}{\left(\lambda\cdot n_{ITO}\right)^{2}+4\cdot\pi\cdot\alpha_{ITO}^{2}-\lambda^{2}\cdot n_{g}}\right)-\frac{4\cdot\pi\cdot n_{ITO}\cdot d_{ITO}}{\lambda}$$

α_{ITO}, n_{ITO} and n_g are the conductive layer absorption coefficient and the refractive indices of the conductive layer and glass, respectively. d_{ITO} and λ denote the conductive layer thickness and the wavelength, respectively.

After to determine the optical parameter of the display elements according to the way described above, the optical parameters of the liquid crystal layer can be determine. In this work the LC layer has dichroic properties, which can be obtained by the measurement of the light transmission through the quasi-ideal planar layer. The first measurements are done for the linearly polarised light according to the layer director, second for perpendicular light polarisation direction. These measurements have to done in two-beams spectrophotometer, when in second beam the same cells but without the liquid crystal is located. Certainly, the calculations of the LC absorption coefficients have to take into account different reflection level from the internal cell boundaries, i.e. air-conductive layer or air-LC. So, the measurements of the liquid crystal refraction indices and the optical properties of the conductive layer have to done first.

The analysis of the light propagation through the LCD should to start from to determine of the illuminated light form and the methods of the light intensity calculations in any point of the display. In this work the GOA method have been assumed. In this method two co-ordinate systems have been introduced (see Fig. 4). These systems are connected with the LC refractive indices ellipsoid, and on the one liquid crystal boundary these both systems cover each other. Inside the LC layer the system x'y' is always connected with the refractive indices ellipsoid and the system xy is invariable.

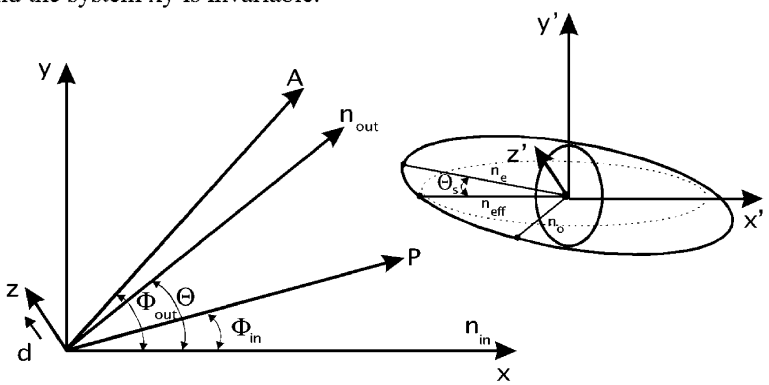

FIGURE 4 The co-ordinate systems assumed in the mathematical model.

n_{in} - director orientation at the input plate (the first infinite layer of LC) overlaps with x axis of the assumed coordinates system;

n_{out} - director orientation at the output plate (the last infinite thin layer), it is changed by an angle Θ in relation to director's orientation at input plate (texture twist angle – for TN effect $\Theta=90^o$);

P – orientation of a polarizer axis, it gives an angle Φ_{in} with x axis;

A – orientation of an analyser axis, it gives an angle Φ_{out} with x axis.

The analysis of the light propagation has been conducted for infinitely thin layers of the display. In such a layer the optical properties are constant. If the all optical parameters have been correctly obtained, the intensity of the light at any point of the display can be calculated. The light has been described in the following form [3,4]:

$$E(x) = A \cdot \sin \omega t + B \cdot \cos \omega t \qquad (3)$$

$$E(y) = C \cdot \sin \omega t + D \cdot \cos \omega t \qquad (4)$$

where in the A, B, C and D coefficients the information about the light intensity and the phase changes are included. In non-dichroic and non-rotated dichroic centres the analysis of the light propagation is very ease and can be done in only xy system In LC layer the both of the system xy and $x'y'$ have to used. Because already second layer can be rotated by an angle $\delta\Theta \to 0$, so the vectors of the light electric field after passing first layer with thickness $\delta z \to 0$ can be described as (according to the Fig. 4) [12]:

$$\begin{aligned} E^1(x') &= [E(x)\cdot\cos\delta\Theta + E(y)\cdot\sin\delta\Theta]\cdot\cos\delta_e \\ E^1(y') &= [-E(x)\cdot\sin\delta\Theta + E(y)\cdot\cos\delta\Theta]\cdot\cos\delta_o \end{aligned} \tag{5}$$

and for the next layers:

$$\begin{aligned} E^n(x') &= [E^{n-1}(x)\cdot\cos\delta\Theta + E^{n-1}(y)\cdot\sin\delta\Theta]\cdot\cos\delta_e \\ E^n(y') &= [-E^{n-1}(x)\cdot\sin\delta\Theta + E^{n-1}(y)\cdot\cos\delta\Theta]\cdot\cos\delta_o \end{aligned} \tag{6}$$

where an upper index denotes number of infinite thin LC layer (thickness $\delta z \to 0$). δ_o represent phase shifts occurring after passing a distance δz in LC medium ($\delta_e = \frac{2\pi n_{eff}\delta z}{\lambda}$ *and* $\delta_o = \frac{2\pi n_o \delta z}{\lambda}$). n_{eff} denotes the effective refractive index of LC and it amounts to:

$$n_{eff} = n_e \cdot n_o \cdot \sqrt{\frac{1+\mathrm{tg}^2\,\Theta_s}{n_o^2 + n_e^2\cdot\mathrm{tg}^2\,\Theta_s}} \quad (\Theta_s - \text{tilt angle}) \tag{7}$$

Certainly, in such a layer the refractive indices of liquid crystals are constant. Using the formulas (3) and (4), the vectors E at any point of the liquid crystal layer is gives as:

$$E^n(x') = A^n\cdot\sin\omega t + B^n\cdot\cos\omega t \tag{8}$$

$$E^n(y') = C^n\cdot\sin\omega t + D^n\cdot\cos\omega t \tag{9}$$

where:

$$A^n = \sqrt{e^{-\alpha_{\parallel}\cdot\delta z}}\cdot\begin{bmatrix} \left(A^{n-1}\cdot\cos\delta\Theta + C^{n-1}\cdot\sin\delta\Theta\right)\cdot\cos\delta_e + \\ +\left(B^{n-1}\cdot\cos\delta\Theta + D^{n-1}\cdot\sin\delta\Theta\right)\cdot\sin\delta_e \end{bmatrix} \tag{10}$$

$$B^n = \sqrt{e^{-\alpha_{\parallel}\cdot\delta z}}\cdot\begin{bmatrix} -\left(A^{n-1}\cdot\cos\delta\Theta + C^{n-1}\cdot\sin\delta\Theta\right)\cdot\sin\delta_e + \\ +\left(B^{n-1}\cdot\cos\delta\Theta + D^{n-1}\cdot\sin\delta\Theta\right)\cdot\cos\delta_e \end{bmatrix} \tag{11}$$

$$C^n = \sqrt{e^{-\alpha_{\perp}\cdot\delta z}}\cdot\begin{bmatrix} \left(-A^{n-1}\cdot\sin\delta\Theta + C^{n-1}\cdot\cos\delta\Theta\right)\cdot\cos\delta_o - \\ +\left(B^{n-1}\cdot\sin\delta\Theta - D^{n-1}\cdot\cos\delta\Theta\right)\cdot\sin\delta_o \end{bmatrix} \tag{12}$$

$$D^n = \sqrt{e^{-\alpha_{\perp}\cdot\delta z}}\cdot\begin{bmatrix} \left(A^{n-1}\cdot\sin\delta\Theta - C^{n-1}\cdot\cos\delta\Theta\right)\cdot\sin\delta_o - \\ +\left(B^{n-1}\cdot\sin\delta\Theta - D^{n-1}\cdot\cos\delta\Theta\right)\cdot\cos\delta_o \end{bmatrix} \tag{13}$$

The coefficients $\alpha_{//}$ and α_{+} denote the absorption indices for the light linearly polarised according to the layer director and perpendicularly to it, respectively.

Appling the methods to determine the optical properties of all display elements and to analyse the light propagation through the display described above, the light intensity can be described in the following formula:

$$I = A^2 + B^2 + C^2 + D^2 \tag{14}$$

PRACTICAL USING OBTAINED RESULTS AND THE POSSIBILITY TO CARRY OUT OF THEM

The theoretical analysis of the light propagation through the LCD and the model of such a display described above have made possible the calculations of the display optical parameters such as: contrast ratio, luminance in on-state and off-state and colour co-ordinates of these both states. In this end the computer program has been worked out. The numerical calculations can be done as the transmission for every light wavelength and as luminance and contrast ratio for all visible range. The luminance and the contrast ratio have been calculated from the following formulas [13,14]:

$$L(\Delta nd) = \frac{\int_{380}^{780} H(\lambda)\cdot V(\lambda)\cdot T(\Delta n, d, \lambda) d\lambda}{\int_{380}^{780} H(\lambda)\cdot V(\lambda) d\lambda} \tag{15}$$

$$CR = \frac{\int_{380}^{780} H(\lambda)\cdot V(\lambda)\cdot T_{ON}(\Delta n, d, \lambda) d}{\int_{380}^{780} H(\lambda)\cdot V(\lambda)\cdot T_{OFF}(\Delta n, d, \lambda) d} \tag{16}$$

where:

- ***H(λ)*** - spectral characteristic of the light source;
- ***V(λ)*** – human eye sensitivity,
- ***T_{ON}(Δn,d,λ), T_{OFF}(Δn,d,λ)*** – display transmission (for internal and external sources) in on-state and off-state, respectively.

The worked out numerical program makes possible to obtain these results as the values or the graphs. The dynamic characteristics have not assumed. The light propagation problem has been solved only for the beam perpendicularly to the display surface. Actually, we are working out this problem for any light beam direction. Additionally, we want to introduce the variable director field.

But, the solved problem make possible to obtain many interesting results. The spectral characteristic for different optical parameters of any display element (see. Fig. 1) can be obtained. Additionally, these characteristic can be done for any dichroic properties of the LC layer, different lighting sources (external and internal, independently), different human eye sensitivities ect.. Such possibility of this program make possible to carry out (very fast) many researches, which give the answer for the question:

1. What do the optical properties of the individual display elements have an influence on the given display parameter in different operation conditions?
2. What the parameters have these elements to have for the given application?

The hypothetical results obtained from this computer program are presented below. Certainly, the possibilities of this program are much better.

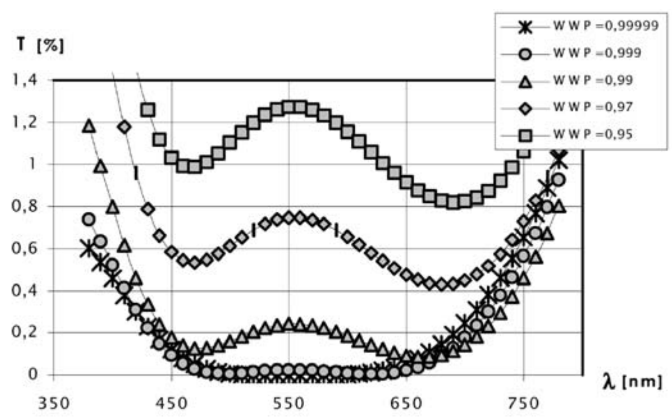

FIGURE 5 Spectral characteristics of a TN layer for reflective system and for *I* transmission minimum (Δnd=0,48). The polarising films with different polarization coefficient WWP have been used.

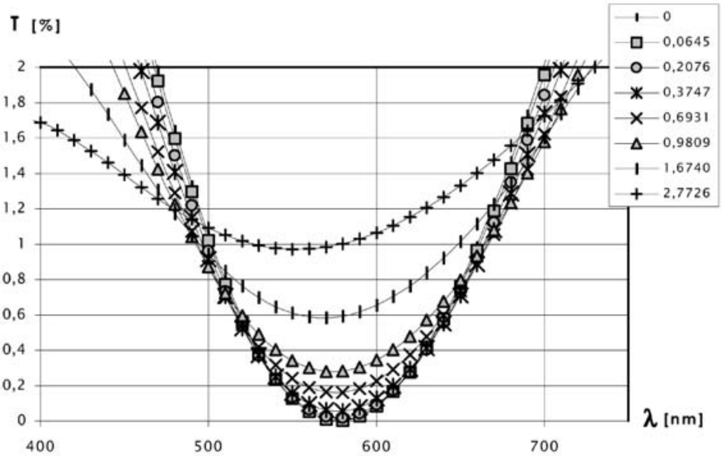

FIGURE 6 The spectral characteristics of a TN layer for transmissive system and *I* minimum (Δnd=0,50). The ideal polarising films have been used. The characteristics have been done for different values of dichroic layer properties. The value $d(\alpha_{II}-\alpha_{+})$ has been announced in the description. d – LC layer thickness, α_{II} and α_{+} - absorption coefficients.

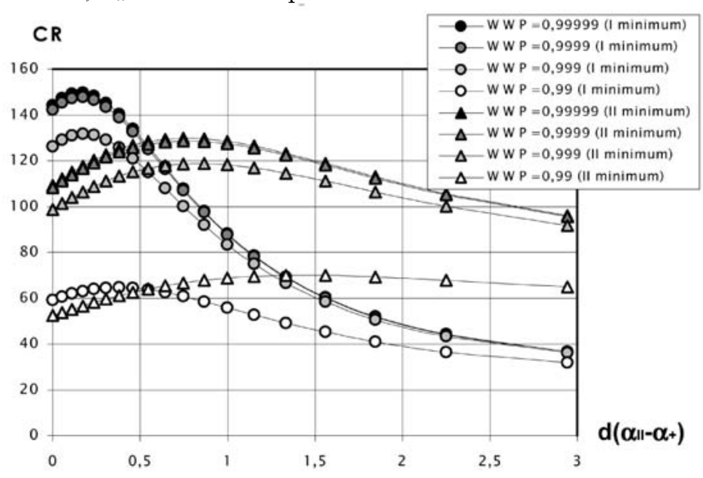

FIGURE 7 The contrast ratio for a TN layer as the function $CR=f(d(\alpha_{II}-\alpha_{+}))$. The graphs have been obtained for transmissive system, *I* and *II* transmission minima and for different polarising films.

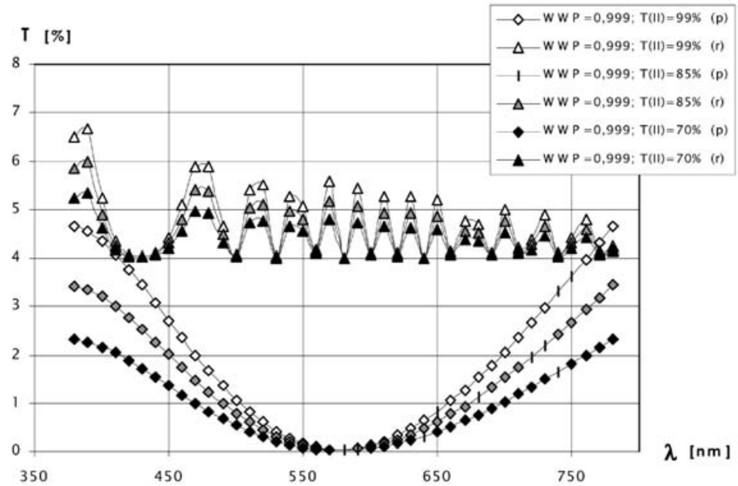

FIGURE 8 The spectral characteristics of the LCD transmission for polarising films with WWP=0,999. The transmissive system has been analysed. External light source *A* and internal light source D_{65} have been assumed. The relationship of the intensities of these light sources have been $I_E/I_I=2$. The symbols *(p)* and *(r)* denote passed beam and reflected beam, respectively.

CONCLUSIONS

The worked out computer program makes possible to calculation the liquid crystal optical parameters such as: contrast ratio, luminance and colour co-ordinates. These calculations can be done for:

- any optical parameters of each elements of the display. Certainly, right measurements of these elements have to be done;
- any the human eye sensitivity;
- any spectral characteristics of the lighting sources (external and internal, independently);
- any tilt and twist angles in the liquid crystal layer.

For a TN effect the calculations have been verified experimentally. The very high conformity of these results has been obtained [11] (see TABLE 1). For this verification the liquid crystal mixtures from MERCK Ltd. have been used. The measurements of the optical parameters of used polarising films and the other elements have been done by two-beams spectrophotometer *BECKMAN*. The cells with thickness about 6μm for a TN effect have been applied.

TABLE 4 Results of calculations and experimental measurements of chosen optical parameters

Mixture	Luminance in off-state L_{OFF} [%]		Luminance in on-state L_{ON} [%]		Effective contrast ratio CR_{obs}		Relative difference [%]		
	Calcul.	Measu.	Calcul.	Measu.	Calcul.	Measu.	L_{OFF}	L_{ON}	CR_{obs}
MLC-6657-100	2,618	2,598	33,68	33,00	12,9	12,7	0,7	2,1	1,6
MLC-6694-000	2,087	2,093	33,68	33,28	16,1	15,9	0,3	1,2	1,3
MLC-13200-000	2,092	2,081	33,68	33,51	16,1	16,1	0,5	0,5	0,0

The obtained results have confirmed that the numerical modelling of the LCD operation can be applied successfully. This modelling can allow easier, faster and cheaper improvement of the display parameters.

ACKNOWLEDGEMENTS
Presented work has been supported by Polish Committee for Scientific Research under grant no. 7 T08A 016 21 and Statutory Activity Support Grants no. PBS 637/WAT/01 and PBW 786/WAT/01.

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